In view of the requirement of the integrability of $F(y)$ at $y=1$ no complementary functions along the lines indicated in (74) appear in this problem.

The integrations in (84) are straightforward when X is taken as a new variable. If we write

$$y_n = \int_0^\infty X^p(1+X)^{-n}dX,$$

then $y_1 = -\pi$ cosec πp, and the recurrence relation $y_n = y_{n-1}(n-2-p)/(n-1)$ is readily obtained. The integral

$$\int_0^\infty \frac{X^p}{X-Y}dX = -Y^p\pi \cot \pi p$$

on taking $\xi = X/Y$ as new variable. These results suffice to integrate (84) when $f(x)$ is of the form specified in (71) and p is suitably chosen.

It is readily verified that $\int_{-1}^{1} F(x)dx$ is zero, confirming the absence of any other complementary function.

The integrations involved in the determination of A and $1+R$ involve the integrals

$$x_n = \int_0^\infty X^p(1-X)^n(1+X)^{-n-2}dX$$

which can be shown to satisfy

$$(n+1)x_n + 2px_{n-1} - (n-1)x_{n-2} = 0$$

x_0 and x_1 are readily expressed in terms of y_2 and y_3 of the previous paragraph, whence the value of x_n follows.

Finally, (62) of the text for $F(x)$ is found on collection of terms, Its integration to give $E(\theta)$ is elementary.

The solution to (70) with completely arbitrary functions $a(x)$ and $b(x)$ has not yet been determined by this method, and it is possible that in this general case more powerful mathematical tools are required. In particular, it has not been possible to find the solution when the radical takes the more complicated form appropriate to the presence of a diaphragm, except in the very special case $\alpha = 1(\mu_2 = 0)$.

A Dielectric Surface-Wave Structure: the V-Line*

P. DIAMENT†, S. P. SCHLESINGER†, MEMBER, IRE, AND A. VIGANTS†

Summary—**Properties of the V-line, a wedge-shaped surface-wave structure comprising a cylindrical dielectric binding medium of sectorial cross section supported by two conducting plates, are considered in terms of its higher-order hybrid modes of propagation. Practical modifications of the ideal structure are emphasized.**

Design curves and equations are presented to determine various propagation parameters and their significance is discussed. Experimental verification of the theory is described.

Introduction

An analysis of surface-wave propagation on dielectric cylinders of sectorial cross section bounded by conducting plates, as in Fig. 1, leads to the usual set of low-order transverse modes and higher-order hybrid modes. In cases of practical interest, however, the prototype structure, here designated "V-line," will be modified in that the plates will be insulated at the apex, whereupon the transverse modes are eliminated and consideration of high-order hybrid modes is required. This modification of the V-line enhances its versatility; in particular, it facilitates the excitation of the modes and permits the application of biasing potentials between the conducting plates. With the use of ferroelectric cylinders, such bias fields may provide convenient electronic control of propagation characteristics.

Although the angle included by the plates is, in principle, unrestricted, for simplicity it will be taken to be an aliquot portion of a semicircle, *i.e.*, π/n radians, where the integer n designates the order of the mode. For such angles, the modes that may be supported by the V-line may propagate on full circular dielectric cylinders as well. The latter waveguide has undergone extensive analysis with respect to its dominant modes.[1–6]

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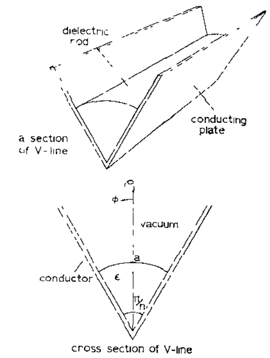

Fig. 1—Geometry and coordinate system for the V-line.

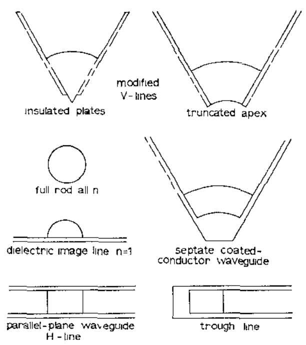

Fig. 2—Modifications of the V-line and some related structures.

The modes of interest for the modified V-line correspond to the higher-order hybrid modes of circular cylinders, which have received only modest attention in the literature.[2,7–9] It will be demonstrated that such higher-order hybrid modes can propagate on a V-line with insulated plates or truncated apex (see Fig. 2) with negligible perturbation.

Structures closely related to the V-line are shown in Fig. 2 and include the septate coated-conductor waveguide,[10] which may be considered a truncated V-line with the apex replaced by a conductor. The dielectric image line[11,12] corresponds to a V-line with $n=1$. The parallel-plane waveguide[13,14] has insulated conducting plates that permit biasing of the binding medium and has fields on both sides of the dielectric. The trough-line[15] has a unilateral field pattern and a continuous conducting boundary. The V-line, as modified at the apex, combines the features of confining the fields to one side of the dielectric and permitting biasing potentials across the binding medium.

Following a brief review of the theory of propagation on the V-line, design curves will be presented for the principal modes of orders two and six for a variety of dielectric constants. Graphs obtained by numerical solution of the equations will be given to illustrate some theoretical results. Experimental work verifying the existence of the high-order modes and their characteristics will be described, together with a discussion of the effects of the separation of the plates.

Theoretical Results

The V-line is a surface-wave structure comprising two conducting plates of infinite extent forming the sides of a wedge and containing within the "V" a sectorial section of dielectric circular cylinder of relative permittivity ϵ, from the apex out to a radius a, with free space in the region beyond the cylinder (see Fig. 1). The line is taken to be infinitely long and is assumed lossless.

With n integral, the modes of propagation on the V-line are also those of a full round rod. For the latter waveguide the HE_{11} mode is dominant, but it can not propagate on a V-line for which $n>1$. The TM_0 mode with axial electric fields is clearly precluded by the longitudinal plates of the V-line. It would appear, then, that the TE_0 mode is the principal mode for the V-line. However, this mode requires the plates to be joined at the apex to accommodate the radial current flow, while practical forms of the V-line have insulated plates or even a truncated apex (see Fig. 2). Thus, the principal mode for the modified V-line is a hybrid mode, in fact, the HE_{n1} mode.

The derivation of the electromagnetic fields for the circular symmetry of the V-line is straightforward.[2,7,8] The axial fields have a sinusoidal angular variation and a Bessel function radial dependence of the form $J_n(p\rho/a)$ inside the dielectric cylinder and $K_n(q\rho/a)$ outside, where the eigenvalues p and q are related to the free-space wavelength λ_0, by

$$p^2 + q^2 = R^2, \qquad R = (2a/\lambda_0)\pi(\epsilon - 1)^{1/2}. \qquad (1)$$

The boundary conditions provide another relation between p and q through the characteristic equation involving Bessel functions and their derivatives given by Schelkunoff and others.[2,1,4]

The eigenvalues p and q conveniently characterize the mode of propagation, not only by supplying the arguments of the Bessel function factors in the field equations, but by yielding much direct information.

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Thus, the guide wavelength λ_g is given by

$$p^2 + \epsilon q^2 = U^2R^2, \qquad U = \lambda_0/\lambda_g. \tag{2}$$

These equations permit mathematical characterizations of physical phenomena. Cutoff of a mode, with equality of guide and free-space wavelengths, *i.e.*, $U=1$, translates into $q=0$ by combining relations (1) and (2). The limiting guide wavelength, $\lambda_g=\lambda_0\epsilon^{-1/2}$ or $U^2=\epsilon$, is describable by $q\rightarrow\infty$ from (2) for finite p. The range of q from 0 to ∞, hence, corresponds to the physical range of guide wavelength from λ_0 to $\lambda_0\epsilon^{-1/2}$ on the V-line. Thus, an analysis of the characteristic equation relating p and q with ϵ and n as parameters is indicated.

A convenient form of the characteristic equation, from which derivatives of Bessel functions are absent, is

$$(J^+ + K^+)(\epsilon J^- - K^-) + (J^- - K^-)(\epsilon J^+ + K^+) = 0 \tag{3}$$

where

$$J^- = \frac{J_{n-1}(p)}{pJ_n(p)}, \qquad J^+ = \frac{J_{n+1}(p)}{pJ_n(p)},$$

$$K^- = \frac{K_{n-1}(q)}{qK_n(q)}, \qquad K^+ = \frac{K_{n+1}(q)}{qK_n(q)}.$$

The cutoff values of p are obtainable by letting $q\rightarrow 0$ in this equation. Using the appropriate limiting process, the solutions are, for $n>1$, the cutoff equations

$$J^- = \frac{1}{(n-1)(\epsilon+1)} \tag{4}$$

$$J_n(p) = 0, \qquad p \neq 0. \tag{5}$$

Eq. (4) was given by Schelkunoff[2] and (5) was reported by the authors.[7,8]

A mathematical analysis of the above characteristic and cutoff equations is available.[7,8] Numerical results will presently be given, for which purpose it will be sufficient to abstract some salient results of the theory.

For $n>1$, the V-line supports a doubly-infinite set of hybrid modes, designated HE_{nm} and EH_{nm}, with cutoff given by (4) and (5), respectively. While q may assume any positive value, increasing with closer binding of the energy to the rod, the range of p for a given mode is confined to that between the roots of $J_{n-2}(p)$ and of $J_{n-1}(p)$ for HE_{nm} modes, and that from the roots of $J_n(p)$ to those of $J_{n+1}(p)$ for EH_{nm} modes. The successive intervals of p defined by these limits correspond to modes of increasing rank m, beginning with the "principal mode" for which $m=1$.

The property of the higher-order hybrid modes of greatest interest here is their negligible field intensity in the vicinity of the apex of the rod. This suggests that the two conducting plates may be insulated at the apex, or the apex truncated, with no appreciable perturbation of the field configuration inside the rod, as will be shown quantitatively.

Numerical Results

The V-line's parameters of interest are the order of the mode n, the permittivity of the rod ϵ, the radial eigenvalues p and q, the diameter-to-wavelength ratio $2a/\lambda_0$, and the free space-to-guide wavelength ratio $U=\lambda_0/\lambda_g$. These are related through (1)–(3). The cutoff values of p and $2a/\lambda_0$ are of importance and are given by (4), (5), and (1). Properties of interest are cutoff, field extent and effects of truncation of the V-line. Theoretical curves, representing solutions of the characteristic equation (3), coupled with relations (1) and (2), are now presented to relate and compare the above characteristics.

Fig. 3 relates p and q according to (3) for the HE_{21} mode for a wide range of ϵ, including the limiting case of $\epsilon=1$ (no rod). While the full range of p is included, only the low values of q, the range of interest for propagation of a pure principal mode, are covered. The asymptote for all curves is the dashed line at the top. Fig. 4 gives the corresponding curves for the HE_{61} mode. The dashed line is again the asymptote for all curves, although the curves for high ϵ do not begin to approach it in the narrow range of q presented.

Fig. 5 relates the parameters $1/U=\lambda_g/\lambda_0$ and $2a/\lambda_0$ for the HE_{21} and HE_{61} modes for various ϵ, and includes the HE_{11} mode for $\epsilon=2.56$ and 8 for comparison. The asymptotes are $\lambda_g/\lambda_0\rightarrow\epsilon^{-1/2}$. The striking contrast in the slopes of the curves for large ϵ is noteworthy; it indicates that large changes in guide wavelengths for small changes in frequency are obtainable near cutoff, or that frequency stability is of increasing importance with increasing ϵ.

The cutoff values of p and $2a/\lambda_0$ for the HE_{21} and HE_{61} modes as functions of ϵ are plotted in Fig. 6. The two curves are related by $(2a/\lambda_0)_c=p_c/\pi(\epsilon-1)^{1/2}$, where the subscript c designates cutoff.

Comparative information on the field extent in V-lines of different angles may be obtained from Fig. 7, which indicates the attenuation in db of the field intensity, relative to that at the surface of the rod, at different distances above the rod. The attenuation ratio is $K_n(q\rho/a)/K_n(q)$. It is clear that the binding of the energy to the rod is greater, the larger the value of n. Proper sizes for the supporting plates of the V-line may also be determined from Fig. 7, which will yield an estimate of the radial extent to which the boundary condition provided by the conducting plates is required.

The possibility of truncating a sizable portion of the V-line without seriously affecting the theory of the full V-line is indicated by Fig. 8. The argument of the Bessel functions in the expressions for the field intensities inside the rod is $p\rho/a$. The 100 per cent curve gives the values of this argument at which maximum intensities are attained. The other curves then give the arguments up to which the intensities are less than the indicated percentage of the maximum intensity. Thus, the curves

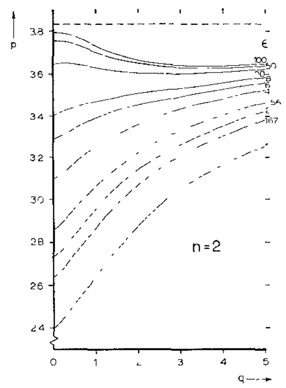

Fig. 3—Solutions of characteristic equation for HE_{21} mode.

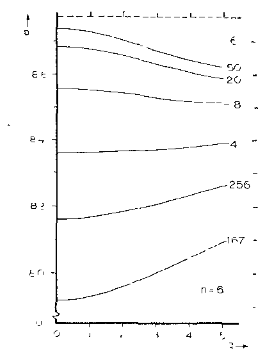

Fig. 4—Solutions of characteristic equation for HE_{61} mode.

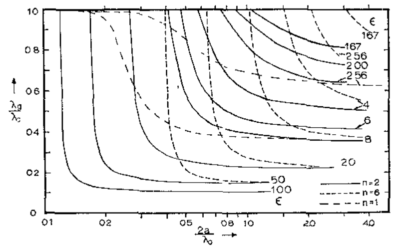

Fig. 5—Dependence of λ_g/λ_0 on $2a/\lambda_0$ for HE_{11}, HE_{21}, and HE_{61} modes for various ϵ.

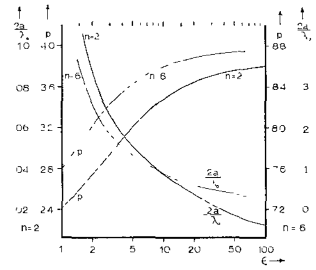

Fig. 6—Cutoff values of p and $2a/\lambda_0$ vs ϵ for HE_{21} and HE_{61} modes.

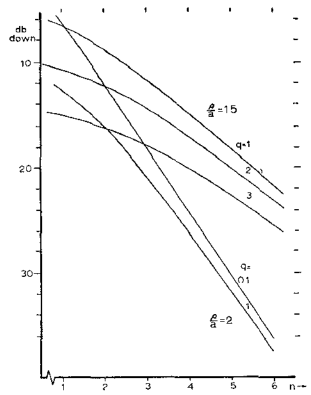

Fig. 7—Field extent vs order of mode. Attenuation in db down from field intensity at surface of rod.

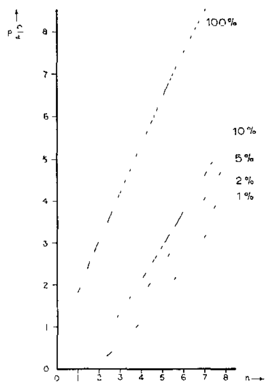

Fig. 8—Truncation of V-line. Radial extent of negligible field intensity vs order of mode. Normalized radius to indicated percentage of maximum field intensity inside rod.

indicate, for example, that for an $n=6$ V-line, as much as 2.42/7.50, or about 30 per cent, of the full radius of the rod may be truncated with loss of field intensity of no more than 1 per cent of the maximum intensity within the rod. The above assumes that the point at which the Bessel functions reach their maximum value is within the rod, *i.e.*, that the operating value of p is greater than the value of $p\rho/a$ indicated by the 100 per cent curve. If this is not so, the curves must be modified in an obvious way to take account of the maximum field intensity actually attained within the rod. It is evident that V-lines of larger n permit the truncation of larger portions of the rod in which the field intensities are negligible. The significance of this is that the two plates supporting the rod may be separated at the apex without seriously affecting the field configuration of the V-line.

It should be emphasized that the curves discussed above, obtained with the aid of an IBM 650 computer, represent only the principal modes of the indicated orders. Similar information for higher-rank modes may be obtained through a more complete solution of the characteristic equation.

Experimental Results

Experimental verification of the theory of propagation of higher-order modes of dielectric cylinders on a V-line was obtained for the HE_{21} mode. The purpose of the experiments was to verify the existence of the higher-order modes, to launch and identify these modes, to measure such properties of the V-line as field extent and insertion loss, and to demonstrate that the modified V-line eliminates the lower-order transverse modes while sustaining the higher-order modes of the dielectric rod waveguide.

The scheme used to verify the existence of, and identify, the HE_{21} mode was to construct the V-line in the form of a resonator,[12] *i.e.*, with transverse conducting plates at both ends of the line, and to predict from the geometry of the V-line the frequencies at which the cavity would be resonant. Several rod sizes and materials were used and various methods of launching and detecting the modes were tried. Experiments were carried out at 3-cm wavelengths.

Theory and experiment were correlated successfully in the case of the 90° V-line with the HE_{21} mode. An effective means of transition from rectangular waveguide to V-line was found in a vertical probe penetrating a short distance into the rod at the apex of the upward-opening V, the supporting plates being separated at the bottom to accommodate the probe at the center of the V-line cavity. The perturbation introduced by insulating the plates at the apex was investigated and compared with the effects on the transverse TE_0 mode. Work is continuing on the excitation of higher-order modes on the V-line, particularly in the case of the HE_{61} mode.

The HE_{21} mode was launched on a 90° V-line with a polystyrene rod ($\epsilon=2.56$) of radius 0.635 in. The resonator was 23.59 in long, *i.e.*, about 25 guide wavelengths long at X band. Coupling to the V-line was through an iris in the resonator end-plate with a tapered transition-piece of the rod inserted into the rectangular waveguide. The resonant frequencies were predicted by determining at which frequencies the length L of the line would be an integral number of half-wavelengths, *i.e.*, when $(L/a)(2a/\lambda_0)U$ is an integer. The results are presented in Table I.

The field extent was checked against the theory by introducing a short probe through a number of holes in one end-plate at various heights above the rod surface in order to compare the axial field intensities. The results of this test are included in Table I.

The experiment was repeated, with some refinements, for a rod of smaller radius ($a=0.507$ in). The results of the resonance tests are plotted in Fig. 9 semilogarithmically in order to segregate the physical dimensions of the V-line from the theoretical parameters; that is, an error in the measurement of L/a would appear on the plot as a shift of the experimental curve parallel to the theoretical curve, since it is a simple factor in $L/(\lambda_g/2)=(L/a)(2a/\lambda_0)U=$integer. The field extent measurements for this case, given in Table II, show poorer agreement with theory, the wave being more loosely bound than was predicted.

To verify the theoretical indications that a separation of the plates at the apex would perturb high-order modes only negligibly while effectively eliminating the transverse electric zero-order mode, the effects of such separation on the resonances of the HE_{21} and TE_{01} modes in a V-line resonator were compared. Virtually no effect was observed upon the HE_{21} resonances throughout X band, while radical changes in the resonant frequencies occurred when the TE_{01} mode was preferentially launched and the conductivity of the apex destroyed. This confirms that the modified V-line is incapable of supporting the transverse electric mode, while the conductivity of the apex is relatively inconsequential for higher-order modes.

TABLE I

Experimental Verification of Resonances and Field Extent in V-Line Resonator

Rod: $n=2$ $\epsilon=2.56$ $a=0.635$ in

Resonant Frequencies

Predicted (kMc)	Experimental (kMc)
9.243	9.244
9.375	9.373
9.508	9.502
9.64	9.639

Field Extent $2a/\lambda_0=1.037$

Probe Position (Reference)	Attenuation	
	Predicted (db)	Experimental (db)
1	0	0
2	12	14.2
3	21.9	23.1

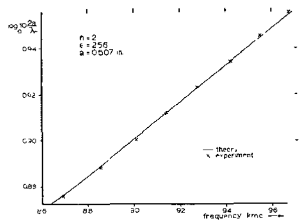

Fig. 9—Verification of support of HE_{21} mode on 90° V-line resonator with polystyrene rod ($\epsilon=2.56$, radius$=0.507$ in). Theoretical and experimental guide wavelength vs frequency.

TABLE II

Verification of Field Extent in V-Line Resonator

Rod: $n=2$ $\epsilon=2.56$ $a=0.507$ in $2a/\lambda_0=0.761$

Field Extent

Probe Position (Reference)	Attenuation	
	Predicted (db)	Experimental (db)
1	0	0
2	5.8	3.5
3	9.9	6.5
4	13.6	10.7

Conclusions

This work has demonstrated the feasibility of the V-line as a surface-wave guiding structure supporting higher-order hybrid modes of propagation. Various characteristics of such modes have been discussed qualitatively on the basis of algebraic solutions of the characteristic equation and, more quantitatively, on the basis of numerical solutions obtained for a variety of orders and dielectric constants.

Experimental confirmation of the theory has been successful qualitatively and quantitatively for the second-order principal mode. Verification has been obtained for the exclusion of the transverse electric mode from the V-line with insulated plates, thereby confirming that it is the high-order hybrid modes rather than the simpler low-order transverse modes that are of interest for operation with disjoint plates. The evitability of joining the plates affords distinct advantages in launching and detecting the modes and in the versatility of the V-line.

The V-line configuration with separated plates or truncated apex appears well suited to convenient electronic control of propagation characteristics through the use of a ferroelectric binding medium. The plates serve as supports for the structure, as image surfaces for the guided wave, and as electrodes for the application of bias potentials. The V-line is distinguished from some other configurations that permit bias fields across a binding medium in that it confines the propagating or radiating fields to one side of the dielectric. The region of the apex remains available for auxiliary structures, such as exciting or detecting mechanisms.

Wave Propagation in a Medium with a Progressive Sinusoidal Disturbance*

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Summary—**A recent paper by Simon derives approximate results, employing only three space harmonics, for the propagation characteristics of an electromagnetic wave traveling in a medium possessing a progressive sinusoidal disturbance. A rigorous result is presented here for this same problem, taking into account all of the space harmonics; also, a sufficiency condition for the convergence of this solution is discussed. This sufficiency condition is not satisfied in a particular case treated by Simon. It is shown that his analysis of this case is in error, and that the total field is singular there. The singular nature of the field is associated with "supersonic" effects in the medium containing the progressive disturbance.**

Introduction

A STIMULATING, recent paper[1] by Simon presents solutions for the propagation characteristics of an electromagnetic wave traveling in a medium possessing a progressive sinusoidal disturbance. This disturbance is expressed in terms of a time-varying dielectric constant, in the form

$$\epsilon = \epsilon_0 + \epsilon_1 \cos(\omega_1 t - k_1 z), \qquad (1)$$

using the notation employed by Simon. The electromagnetic field is then expanded in terms of spatial harmonics and a relation is found between the amplitudes of these space harmonics. This relation is essentially a system of infinite homogeneous equations with an infinite number of unknowns. Simon then points out that in problems of interest ϵ_1 is very small, and that a rigorous solution to this system of equations is not simple to obtain. He, therefore, adopts a perturbation approach, and retains only the lowest three of the infinite number of space harmonics. With this approximation, he obtains a determinantal equation for the propagation constants, solves this equation for several interesting special cases, and then obtains the corresponding space-harmonic amplitudes.

A major contribution of Simon's paper lies in the stress he places on the interrelation between physical concepts in different disciplines. For example, while it has long been known that a stop band for electromagnetic waves in a periodic structure corresponds to Bragg reflection in crystals, Simon relates the Doppler effect produced in a stop band associated with a moving disturbance to parametric amplification phenomena. The conditions for both up-conversion and down-conversion are considered in some detail, and approximate expressions are presented for the propagation constants and the fields. An additional so-called "triple root" case is also treated in some detail, but the results are of questionable value, for reasons presented below.

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† Microwave Research Inst., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y.

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